

Generalized $su(2)$ coherent states for the Landau levels and their nonclassical properties

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Abstract

Following the lines of the recent papers [J. Phys. A: Math. Theor. 44, 495201 (2012); Eur. Phys. J. D 67, 179 (2013)], we construct here a new class of generalized coherent states related to the Landau levels, which can be used as the finite Fock subspaces for the representation of the $su(2)$ Lie algebra. We establish the relationship between them and the deformed truncated coherent states. We have, also, shown that they satisfy the resolution of the identity property through a positive definite measures on the complex plane. Their nonclassical and quantum statistical properties such as quadrature squeezing, higher order ‘ $su(2)$ ’ squeezing, anti-bunching and anti-correlation effects are studied in details. Particularly, the influence of the generalization on the nonclassical properties of two modes is clarified.

Keywords: Nonlinear Coherent States, Sub-Poissonian Statistics, Squeezing Effect, Landau Levels.

1 Introduction

Coherent states (CSs), were first established by Schrödinger [1] as the eigenvectors of the boson annihilation operator, \hat{a} , corresponding to the Heisenberg-Weyl Lie algebra. They play an important role in quantum optics and provide us with a link between quantum and classical oscillators. Moreover, these states can be produced by acting of the Glauber displacement operator, $D(z) = e^{z\hat{a}^\dagger - \bar{z}\hat{a}}$, on the vacuum states, where z is a complex variable. These states were later applied successfully to some other models based on their Lie algebra symmetries by Glauber [2, 3], Klauder [4, 5], Sudarshan [6], Barut and Girardello [7] and Perelomov [8]. Additionally, for the models with one degree of freedom either discrete or

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continuous spectra- with no remark on the existence of a Lie algebra symmetry- Gazeau et al proposed new CSs, which were parametrized by two real parameters [9, 10]. Moreover, there exist some considerations in connection with CSs corresponding to the shape invariance symmetries [11, 12]. To construct CSs, four main different approaches the so-called Schrödinger, Klauder-Perelomov, Barut-Girardello, and Gazeau-Klauder methods have been found, so that the second and the third approaches rely directly on the Lie algebra symmetries and their corresponding generators. Here, it is necessary to emphasize that quantum coherence of states nowadays pervade many branches of physics such as quantum electrodynamics, solid-state physics, and nuclear and atomic physics, from both theoretical and experimental viewpoints.

In addition to CSs, squeezed states (SSs) have attracted much attention during past decades. These are non-classical states of the electromagnetic field in which certain observables exhibit fluctuations less than the vacuum state [13]. These states are interesting because they can achieve lower quantum noise than the zero-point fluctuations of the vacuum or coherent states. Over the last four decades there have been several experimental demonstrations of nonclassical effects, such as the photon anti-bunching [14], sub-Poissonian statistics [15, 16], and squeezing [17, 18]. Also, considerable attention has been paid to the deformation of the harmonic oscillator algebra of creation and annihilation operators [19]. Some important physical concepts such as the CSs, the even- and odd-CSs for ordinary harmonic oscillator have been extended to deformation case. Moreover, there exist interesting quantum interference effects related to the quantum states that are namely superposition states, too [20, 21]. Besides, superpositions of CSs can be prepared in the motion of a trapped ion [22, 23]. With respect to the nonclassical effects, the coherent states turn out to define the limit between the classical and nonclassical behavior.

Another type of generalization of CSs is the nonlinear coherent states (NLCSs), or f-CSs [24]. They are associated with nonlinear algebras and defined as the eigenstates of the annihilation operator of a f-deformed oscillator $f(\hat{N})a$. Indeed, the nature of the nonlinearity depends on the choice of the function $f(\hat{N})$ [25]. These states may appear as stationary states of the center-of mass motion of a trapped ion [26, 27]. NLCSs exhibit nonclassical features such as quadrature squeezing, sub-Poissonian statistics, anti-bunching, self-splitting effects and so on [28-30].

The discrete energy values corresponding to the motion of a charged particle on a flat surface in the presence of a uniform external magnetic field perpendicular to this plane are called Landau levels [31]. The physics of charged particles in the presence of magnetic field has been one of the important problems in quantum mechanics, inspired by condensed matter physics, quantum optics etc. From the classical viewpoint, the magnetic field creates a transverse current perpendicular to both the direction of motion of particle and the direction of the magnetic field. Therefore, the Landau problem can be counted as a cornerstone of quantum Hall effect. In the last three decades, many efforts have been carried out to describe the spectral properties of the quantum Hall effect [32-39]. The quantum Hall effect, as a universal phenomenon, is observed on any two-dimensional surface with a charged particle moving in the presence of a strong perpendicular uniform magnetic field. In metals and other dense electronic systems the electrons occupy many Landau levels. Furthermore, the kinetic energy levels of electrons in two-dimensional gas correspond to Landau levels. For all these reasons, the study of coherent states for Landau levels is of great importance which

was first studied by Malkin and Man'ko [40] and later by many others [41-55].

Recently, we have introduced the generalized coherent states (GCSs) for harmonic and pseudo harmonic oscillators based on the generalization of the bosonic displacement operators associated with the Heisenberg-Weyl and $su(1, 1)$ Lie algebras, respectively [56, 57]. It has been shown that, they are new class of nonlinear coherent states and have some interesting features such as temporal stability and nonclassical properties. An interesting feature of the above mentioned approach is due to the fact that, contrary to the Klauder-Perelomov and Barut-Girardello approaches, they do not require for existence of dynamical symmetries associated with the considered system. In the other word, we need only the raising operator associated with the considered system in the framework of supersymmetric quantum mechanics.

Along with extension of the above scenarios to the $su(2)$ Lie algebra counterpart, we introduce generalized $su(2)$ CSs for the problem of an electron moving in the constant magnetic field. These states admit a resolution of the identity through positive definite and non-oscillating measures on the complex plane. We have shown that these states are nonlinear truncated coherent states with a spacial nonlinearity function. It has been discussed, in details, that they have indeed nonclassical features such as squeezing effect and sub-Poissonian statistics. The results are compared with the properties of the well known $su(2)$ coherent states [58].

This paper is organized as follows: in section 2, we briefly review on the Schwinger representation of the $su(2)$ Lie algebra symmetry of Landau levels. Section 3 is devoted to construction the new class of generalized $su(2)$ CSs $|z\rangle_r$, via generalized analogue of the displacement operators acting on the Landau levels with lowest z-angular momentum. In order to realize the resolution of the identity, we have found the positive definite measures on the complex plane. There, it has been shown that these states can be interpreted as the nonlinear truncated coherent states. Furthermore, in section 4 by evaluating some physical quantities, we discuss their statistical and nonclassical properties. Finally, we conclude the paper in section 5.

2 Landau levels

Let us first explain the exact solvability of the symmetric-gauge Landau Hamiltonian corresponding to the motion of an electron on a flat surface in the presence of the uniform magnetic field in the positive direction of z -axis, i.e. $H = \hbar\omega(a^\dagger a + 1/2) = \hbar\omega(b^\dagger b + 1/2) - \omega L_3$ with $L_3 = -i\hbar\partial/\partial\varphi$. It has an infinite-fold degeneracy on the Landau levels, that is

$$H |n, m\rangle = \hbar\omega(n + 1/2) |n, m\rangle, \quad (1)$$

in which Landau cyclotron frequency is expressed in terms of the value of the electron charge, its mass, the magnetic field strength B and also the velocity of light as $\omega = eB/Mc$. Here, m is an integer number and n is a nonnegative one together with $n \geq -m$ limitation. Each pair of operators (a, a^\dagger) and (b, b^\dagger) have the following explicit forms in terms of the polar

coordinates $0 < r < \infty$ and $0 \leq \varphi < 2\pi$ for two-dimensional flat surface [53],

$$a = -e^{i\varphi} \sqrt{\frac{\hbar}{2M\omega}} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \varphi} + \frac{M\omega}{2\hbar} r \right), \quad a^\dagger = e^{-i\varphi} \sqrt{\frac{\hbar}{2M\omega}} \left(\frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \varphi} - \frac{M\omega}{2\hbar} r \right), \quad (2)$$

$$b = e^{-i\varphi} \sqrt{\frac{\hbar}{2M\omega}} \left(\frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \varphi} + \frac{M\omega}{2\hbar} r \right), \quad b^\dagger = -e^{i\varphi} \sqrt{\frac{\hbar}{2M\omega}} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \varphi} - \frac{M\omega}{2\hbar} r \right), \quad (3)$$

and form two separate copies of Weyl-Heisenberg algebra,

$$[a, a^\dagger] = 1, \quad [b, b^\dagger] = 1, \quad [a, b^\dagger] = [a^\dagger, b] = [a, b] = [a^\dagger, b^\dagger] = 0, \quad (4)$$

with the unitary representations as

$$a |n, m\rangle = \sqrt{n} |n-1, m+1\rangle, \quad a^\dagger |n-1, m+1\rangle = \sqrt{n} |n, m\rangle, \quad (5)$$

$$b |n, m\rangle = \sqrt{n+m} |n, m-1\rangle, \quad b^\dagger |n, m-1\rangle = \sqrt{n+m} |n, m\rangle. \quad (6)$$

Infinite fold degeneracy via the magnetic quantum number m is a result of existence of rotational symmetry, i.e. $[H, L_3] = 0$. Besides, the infinite Fock subspace corresponding to a given energy represents the dynamical symmetry group of Weyl-Heisenberg: $[H, b] = [H, b^\dagger] = 0$. Landau levels are orthonormal with respect to integration over the entire plane, that is

$$\langle n, m | n', m' \rangle := \int_{\varphi=0}^{2\pi} \int_{r=0}^{\infty} \langle r, \varphi | n, m \rangle^* \langle r, \varphi | n', m' \rangle r dr d\varphi = \delta_{nn'} \delta_{mm'}, \quad (7)$$

in which

$$\langle r, \varphi | n, m \rangle = \sqrt{\frac{n!}{\pi(n+m)!}} \left(\frac{M\omega}{2\hbar} \right)^{m+1} r^m e^{im\varphi} e^{-\frac{M\omega}{4\hbar} r^2} L_n^{(m)}(M\omega r^2 / 2\hbar) \quad (8)$$

is the polar coordinate representation of Landau levels in terms of the associated Laguerre functions. At the end of this section, let us denote the Hilbert space corresponding to all Landau levels with $\mathcal{H} = \text{span}\{|n, m\rangle\}_{n=0}^{\infty, m=-n}^{\infty}$. For a given n the Hilbert space \mathcal{H} can be split into the infinite direct sums of finite dimensional Hilbert subspaces \mathcal{H}_n , i.e. $\mathcal{H} = \oplus_{n=0}^{\infty} \mathcal{H}_n$ with $\mathcal{H}_n = \text{span}\{|n-k, -n+2k\rangle\}_{k=0}^n$ [53].

3 Generalized $su(2)$ CSs for Landau levels

Using the commutation relations of the Weyl-Heisenberg algebras given in Eq.(4), one can easily find that the following generators

$$K_- := a^\dagger b, \quad K_+ := ab^\dagger, \quad K_3 := \frac{b^\dagger b - a^\dagger a}{2}, \quad (9)$$

produce the bosonic and unitary representation of the $su(2)$ Lie algebra with the standard commutation relations

$$[K_+, K_-] = 2K_3, \quad [K_3, K_\pm] = \pm K_\pm. \quad (10)$$

From Eqs. (5) and (6), it becomes obvious that the finite dimensional Hilbert space \mathcal{H}_n represent the $su(2)$ Lie algebra as

$$K_+|n+1, m-2\rangle = \sqrt{(n+1)(n+m)}|n, m\rangle, \quad (11a)$$

$$K_-|n, m\rangle = \sqrt{(n+1)(n+m)}|n+1, m-2\rangle, \quad (11b)$$

$$K_3|n, m\rangle = \frac{m}{2}|n, m\rangle. \quad (11c)$$

Note that K_3 is a self-adjoint operator and the two operators K_- and K_+ are Hermitian conjugate of each other with respect to the inner product (7).

Now, generalized $su(2)$ CSs for an electron in the constant magnetic field are introduced via generalized analogue of the displacement operators acting on the Landau levels with lowest z-angular momentum, $|n, -n\rangle$, as

$$|z\rangle_r := M_r^{-\frac{1}{2}}(|z|)_1 F_r([n-1], [n-1, n, n+1, \dots, n+r-1], zK_+)|n, -n\rangle, \quad r \geq 1, \quad (12)$$

where $z(=|z|e^{i\phi})$ and r are respectively the coherence and the deformation parameters, respectively. Clearly, $|z\rangle_r$ becomes the standard $su(2)$ CSs corresponding to bosonic realization of the $su(2)$ Lie algebra [58], when r tends to unity and z be replaced with $\frac{z}{|z|} \tanh(|z|)$. Using by the series form of the hypergeometric functions and applying the Eq. (11(a)), the states $|z\rangle_r$ can be obtained as

$$|z\rangle_r = M_r^{-\frac{1}{2}}(|z|) \sum_{k=0}^n z^k \left[\prod_{l=1}^{r-1} \frac{\Gamma(n+l-1)}{\Gamma(n+k+l-1)} \right] \sqrt{\frac{n!}{(n-k)!k!}} |n-k, -n+2k\rangle, \quad r \geq 2, \quad (13)$$

where the normalization constant $M_r(|z|)$ is chosen so that $|z\rangle_r$ is normalized to unity with respect to the inner product (7), i.e. ${}_r\langle z|z\rangle_r = 1$, then

$$M_r(|z|) = {}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2). \quad (14)$$

It should be noticed that, these states can be categorized as special class of *Generalized Hypergeometric CSs* [59] have already been made by Appl et al. Using the inner product (7), the overlapping of the generalized $su(2)$ CSs can be calculated as follows:

$${}_r\langle z_1|z_2\rangle_r = \frac{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -\bar{z}_1 z_2)}{\sqrt{M_r(|z_1|)M_r(|z_2|)}}, \quad (15a)$$

$${}_{r_1}\langle z|z\rangle_{r_2} = \frac{{}_1F_{r_1+r_2-2}([-n], [n, n+1, \dots, n+r_1-2, n+r_2-2], -|z|^2)}{\sqrt{M_{r_1}(|z|)M_{r_2}(|z|)}}, \quad (15b)$$

and result that two different kinds of these states are non-orthogonal, if $r_1 \neq r_2, z_1 \neq z_2$. Now, we should check realization of the property of *resolution to identity* for the states $|z\rangle_r$ in the Hilbert space \mathcal{H}_n .

$$1_{\mathcal{H}} = \oint_{\mathbb{C}(z)} d^2z K_r(|z|)|z\rangle_r {}_r\langle z| = \sum_{m=-n}^n |n, m\rangle\langle n, m|, \quad (16)$$

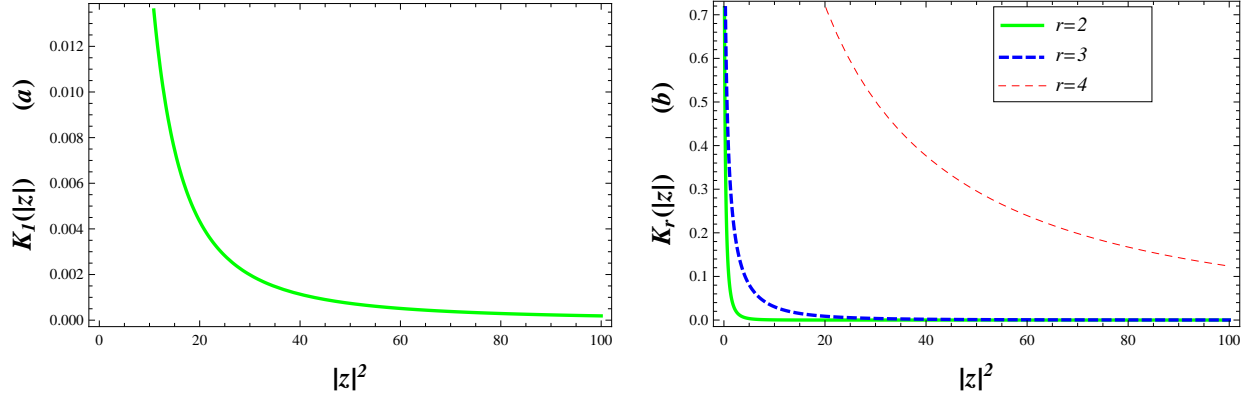


Figure 1: Plots of the positive definite measures $K_r(|z|)$ in terms of $|z|^2$ for different values of r . (a) relates to $r = 1$ and (b) corresponds to higher amounts of $r = 2, 3$ and 4 .

where $K_r(|z|)$ is a positive definite measure to be determined below. By substituting Eq. (13) in Eq. (16) lead us to the following integral relation

$$\int_0^\infty d|z| |z|^{2k+1} \frac{\pi K_r(|z|)}{M_r(|z|)} = \left[\prod_{l=1}^{r-1} \frac{\Gamma(n+k+l-1)}{\Gamma(n+l-1)} \right]^2 \frac{(n-k)!k!}{n!}. \quad (17)$$

Along with the integral relation for the Meijers G-functions (see $\frac{7-811}{4}$ in [60]), the positive definite and non-oscillating measure $K_r(|z|)$ is obtained as

$$K_r(|z|) = \frac{{}_2F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], |z|^2)}{\pi \left[\prod_{l=1}^{r-1} \Gamma(n+l-1) \right]^2} \times G([[-n-1], [], [0, n-1, n-1, \dots, n+r-3, n+r-3], [], |z|^2). \quad (18)$$

For ($r = 1, 2, 3$ and 4) we have plotted the changes of the non-oscillating and positive definite measures $K_r(|z|)$ in terms of $|z|^2$ in Figures 1.

3.1 Generalized $su(2)$ CSs as the deformed truncated CSs

Now, we show that the states $|z\rangle_r$ can be interpreted as deformed truncated CSs associated with f -deformed Schwinger realization of $su(2)$ Lie algebra (10) with a special deformation function. The deformed truncated CSs [61, 62] are defined, in the similar approach of constructing of the truncated CSs [63], as

$$|z, f\rangle = C^{-\frac{1}{2}}(|z|) \exp(z f(\hat{n}_a, \hat{n}_b) a b^\dagger) |n, -n\rangle, \quad (19)$$

where $f(\hat{n}_a, \hat{n}_b)$ is the deformation function which depends on the photon numbers of \hat{n}_1 and \hat{n}_2 . Now by using the following relations

$$a f(\hat{n}_a, \hat{n}_b) = f(\hat{n}_a + 1, \hat{n}_b) a, \quad (20a)$$

$$b^\dagger f(\hat{n}_a, \hat{n}_b) = f(\hat{n}_a, \hat{n}_b - 1) b^\dagger, \quad (20b)$$

we obtain the expansion of the $|z, f\rangle$ as

$$|z, f\rangle = C^{-\frac{1}{2}}(|z|) \sum_{k=0}^n z^k [f(n_a + k - 1, n_b - k + 1)]! \sqrt{\frac{n!}{k!(n-k)!}} |n - k, -n + 2k\rangle \quad (21)$$

where $[f(n_a + k - 1, n_b - k + 1)]! = \prod_{s=0}^{k-1} f(n_a + s, n_b - s)$. Now, by comparing the Eq. (13) with the Eq. (21), one can find that $|z\rangle_r$ can be considered as deformed truncated CSs with the deformation function $f(\hat{n}_a, \hat{n}_b) = \frac{\Gamma(-\hat{n}_a + 2n - 1)}{\Gamma(-\hat{n}_a + 2n + r - 2)}$. Obviously, it tends to unity for $r = 1$.

4 Statistical properties of generalized $su(2)$ CSs

In this section, some quantum statistical properties including squeezing in field quadratures (x and its canonical conjugate p_x), higher order squeezing, photon statistics and cross-correlation of these states are studied in details. They provide appropriate framework for the study of (non-)classical features of these quantum states [64].

\diamond Squeezing in Field Quadratures x and p_x

To investigate the quadrature squeezing in the introduced generalized CSs we now construct the creation and annihilation operators of the system in terms of the coordinates x and its conjugate momenta p_x . From Eqs. (2) and (3), the operators x and p_x can be easily written as

$$x = \sqrt{\frac{\hbar}{2M\omega}}(b + b^\dagger - a - a^\dagger), \quad p_x = \frac{i}{2}\sqrt{\frac{M\hbar\omega}{2}}(b - b^\dagger + a - a^\dagger), \quad (22)$$

which satisfy the commutation relation $[x, p_x] = i\hbar$. From these, the uncertainty condition for the variances of the quadratures x, p_x follow

$$\sigma_{xx}\sigma_{p_x p_x} \geq \frac{\hbar^2}{4}, \quad (23)$$

where $\sigma_{ab} = \frac{1}{2} \langle ab + ba \rangle - \langle a \rangle \langle b \rangle$. To characterize the degree of squeezing in the x and p_x components, we introduce the following parameters [55]

$$S_x = \frac{2\sigma_{xx}}{\sqrt{\hbar^2 + 4\sigma_{xp_x}^2}} - 1, \quad S_{p_x} = \frac{2\sigma_{p_x p_x}}{\sqrt{\hbar^2 + 4\sigma_{xp_x}^2}} - 1. \quad (24)$$

For our generalized $su(2)$ CSs (Eq. (13)) we have

$$\langle a \rangle = \langle b \rangle = \langle a^2 \rangle = \langle b^2 \rangle = \langle ab \rangle = 0, \quad (25)$$

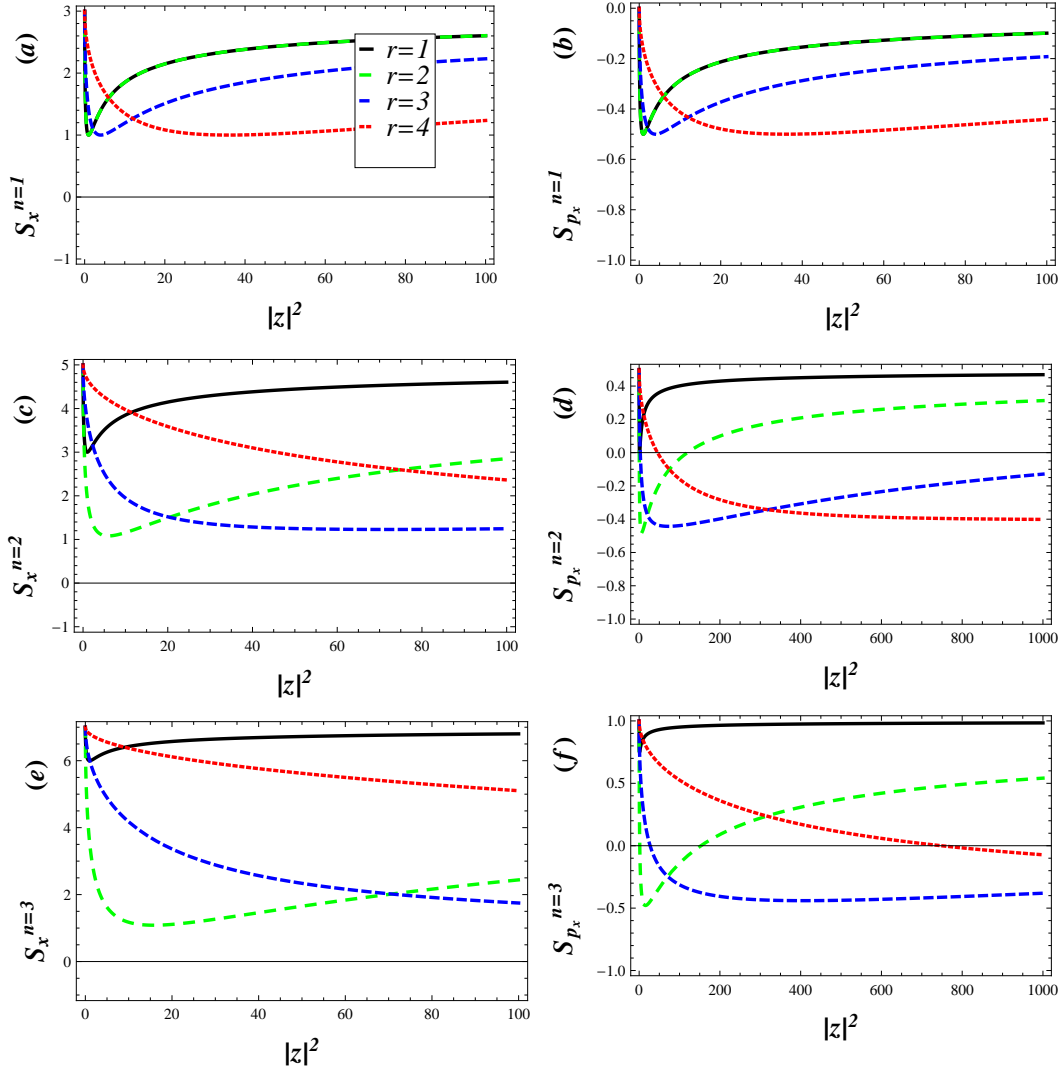


Figure 2: The plots of quadrature squeezing parameter S_x^n (a, c, e) and $S_{p_x}^n$ (b, d, f) respectively, against $|z|^2$ for $\phi = 0$ and different values of n and r .

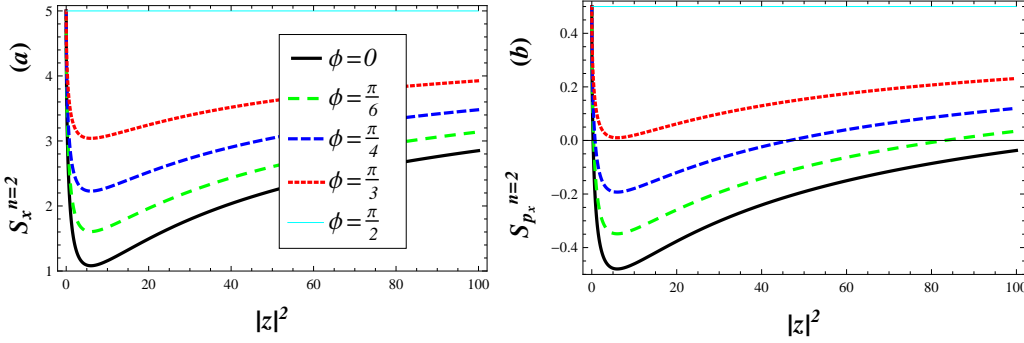


Figure 3: The plots of quadrature squeezing parameter S_x^n (a) and $S_{p_x}^n$ (b) respectively, against $|z|^2$ for different values of ϕ by setting $r = 2$ and $n = 2$.

from which it follows that

$$\left(\frac{M\omega}{\hbar}\right) S_x = 1 + n - 2|z| \times \frac{{}_1F_{2r-2}([1-n], [n, n+1, n+1, \dots, n+r-2, n+r-2, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, n+1, n+1, \dots, n+r-2, n+r-2], -|z|^2)} \cos \phi, \quad (26a)$$

$$\left(\frac{M\omega}{\hbar}\right) S_{p_x} = 1 + n - 2|z| \times \frac{{}_1F_{2r-2}([1-n], [n, n+1, n+1, \dots, n+r-2, n+r-2, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, n+1, n+1, \dots, n+r-2, n+r-2], -|z|^2)} \cos \phi. \quad (26b)$$

Quadrature squeezing in the generalized $su(2)$ CSs exists if S_x or S_{p_x} is in the range $(-1, 0)$. It follows from Eqs. (26) that the quantities S_x and S_{p_x} are dependent on the Landau cyclotron frequency ω which comes from the variations of magnetic field B_{ext} too. In Figure 2, we show S_x and S_{p_x} as functions of $|z|^2$ for $r (= 1, 2, 3 \text{ and } 4)$ as well as $n (= 1, 2 \text{ and } 3)$ for fixed $(\phi = 0, M = \hbar = \omega = 1)$. From this, there is no squeezing in the x component. As shown, for $r = 1$ quadrature squeezing in the p_x component occurs only for $n = 1$. In the other word the well-known $su(2)$ CSs for Landau levels do not exhibit quadrature squeezing for $n > 1$ while, our introduced states have squeezing property for any values of n . Meanwhile, from Figure 2 (b, d, f), by increasing the generalization parameter r , quadrature squeezing hold for large value of $|z|^2$. Also, we show in Figure 3 the squeezing parameter S_x and S_{p_x} for different values of $\phi (= 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \text{ and } \frac{\pi}{2})$ for fixed $(r = 2, M = \hbar = \omega = 1)$. This figure show that quadrature squeezing in the p_x component occur for $\phi < \frac{\pi}{3}$.

◇ Higher Order ‘ $su(2)$ ’ Squeezing

In order to study $su(2)$ squeezing we consider the following Hermitian quadrature operators

$$X_1 = \frac{K_+ + K_-}{2}, \quad X_2 = \frac{K_- - K_+}{2i}, \quad (27)$$

with the commutation relation $[X_1, X_2] = -iK_3$. From this commutation relation the uncertainty relation for the variances of the quadrature operators X_i follows

$$\langle (\Delta X_1)^2 \rangle \langle (\Delta X_2)^2 \rangle \geq \frac{|\langle K_3 \rangle|^2}{4}, \quad (28)$$

where $\langle (\Delta X_1)^2 \rangle = \langle (X_1)^2 \rangle - \langle X_1 \rangle^2$ and the angular brackets denote averaging over an arbitrary normalizable state for which the mean values are well defined, $\langle X_i \rangle = {}_r\langle z | X_i | z \rangle_r$. Following Walls (1983) as well as Wodkiewicz and Eberly (1985) [65, 66] we will say the state is $su(2)$ squeezed if the variance of one of the generators X_i be smaller than the average uncertainty of the quantum-mechanical fluctuations:

$$\langle (\Delta X_i)^2 \rangle < \frac{|\langle K_3 \rangle|}{2}, \quad \text{for } i = 1 \text{ or } 2. \quad (29)$$

To measure the degree of the $su(2)$ squeezing we introduce the squeezing factor S_i [67]

$$S_i = \frac{\langle (\Delta X_i)^2 \rangle - \frac{|\langle K_3 \rangle|}{2}}{\frac{|\langle K_3 \rangle|}{2}}, \quad (30)$$

it leads that the $su(2)$ squeezing condition takes on the simple form $S_i < 0$, however maximally squeezing is obtained for $S_i = -1$. By using of the mean values of the generators of the $su(2)$ Lie algebra, one can derive the variance of the quadrature operators $X_{1(2)}$ as

$$\langle (\Delta X_{1(2)})^2 \rangle = \frac{2 \langle K_+ K_- \rangle - 2 \langle K_3 \rangle \pm \langle K_+^2 + K_-^2 \rangle - \langle K_- \pm K_+ \rangle^2}{4}. \quad (31)$$

With the help of following mean values of the generators of the $su(2)$ Lie algebra with respect to states (13)

$$\begin{aligned} \langle K_+ \rangle &= \overline{\langle K_- \rangle} = \frac{\Gamma(n+1)}{\Gamma(n+r-1)} \bar{z} \\ &\times \frac{{}_1F_{2r-2}([1-n], [n, n+1, n+1, \dots, n+r-2, n+r-2, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2)}, \\ \langle K_+^{\lambda^2} \rangle &= \overline{\langle K_-^{\lambda^2} \rangle} = \left(\frac{n-1}{n+r-1} \right) \left(\frac{\Gamma(n+1)}{\Gamma(n+r-1)} \right)^2 \bar{z}^2 \\ &\times \frac{{}_5F_{2r+2}([2-n, n, n+1, n+r-1, n+r], [n, n, \dots, n+r-1, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2)}, \\ \langle K_+ K_-^{\lambda} \rangle &= \left(\frac{\Gamma(n+1)}{\Gamma(n+r-1)} \right)^2 |z|^2 \\ &\times \frac{{}_2F_{2r-1}([1-n, 1-n], [-n, n+1, n+1, \dots, n+r-1, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2)}, \\ \langle K_3 \rangle &= - \left(\frac{n}{2} \right) \frac{{}_2F_{2r-1}([-n, 1-\frac{n}{2}], [-\frac{n}{2}, n, n, \dots, n+r-2, n+r-2], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2)}, \end{aligned} \quad (58)$$

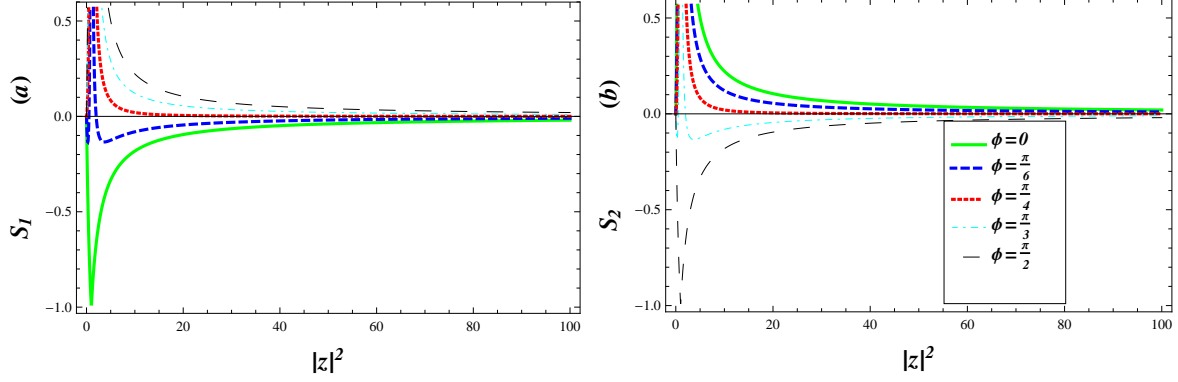


Figure 4: The plots of $su(2)$ squeezing parameter (a) S_1 and (b) S_2 versus $|z|^2$ for different values of ϕ and $n = 1$ for the standard $su(2)$ CSs for Landau levels.

the squeezing quantities $S_{1(2)}^n$ can be easily evaluated for states $|z\rangle_r$.

In figure (4), we plot the $S_{1(2)}^n$ as a function of $|z|^2$ for standard two-mode CSs. As shown in figures 4(a) and (b), the curves of S_1 and S_2 corresponding to squeezing in the X_1 and X_2 operators indicate that for $n = 1$, there is a quadrature squeezing effect in the both X_1 and X_2 components. From this figure, squeezing effect occurs in the $X_1(X_2)$ component for $\phi = \frac{\pi}{6}(\phi = \frac{\pi}{4})$ in the major range of $|z|^2$, while for $\phi = 0(\phi = \frac{\pi}{2})$ the squeezing parameters $S_1(S_2)$ are negative in all the range of $|z|^2$. Also, in limit of $|z|^2 \rightarrow 1$, maximal squeezing in the X_1 and X_2 components occur for $\phi = 0$ and $\phi = \frac{\pi}{2}$, respectively.

In figure (5), we plot the S_1 and S_2 against $|z|^2$ for different values of $r(= 2, 3$ and $4)$ as well as $n(= 1, 2, 3)$ with respect to generalized $su(2)$ CSs, $|z\rangle_r$. Here, we choose the phase $\phi = 0$. From this figure, the quadrature squeezing occur only in the X_1 component for different values of n . As shown in figure (a), for $n = 1$ the squeezing parameter S_1 is negative in all the range of $|z|^2$. This indicate that for $n = 1$ and $\phi = 0$, the states $|z\rangle_r$ are squeezed states. Also, we show in Figures (6) the squeezing parameter S_1 and S_2 for different values of $\phi(= 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3},$ and $\frac{\pi}{2})$ for fixed $r = 3$. This figure show that while quadrature squeezing in the X_1 component occur for $\phi = 0$ and $\phi = \frac{\pi}{6}$, there is quadrature squeezing effect in the X_2 component for $\phi = \frac{\pi}{3}$ and $\phi = \frac{\pi}{2}$.

◇ *Photon statistics*

Now we are in a position to study the photon statistics of the constructed generalized $su(2)$ CSs given by Eq. (13). For this purpose, we calculate the Mandel parameter of bosonic

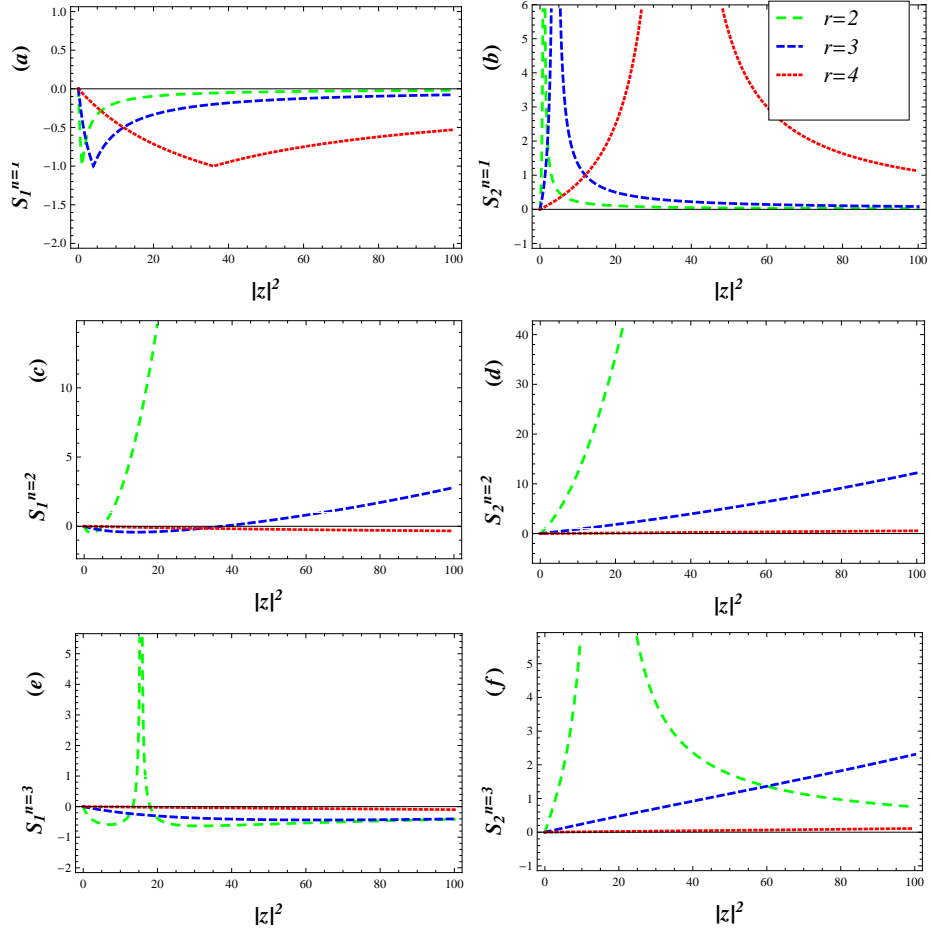


Figure 5: The plots of quadrature squeezing parameter S_1^n (a, c, e) and S_2^n (b, d, f), respectively versus $|z|^2$ for different values of r as well as different n while we choose the phase $\phi = 0$.

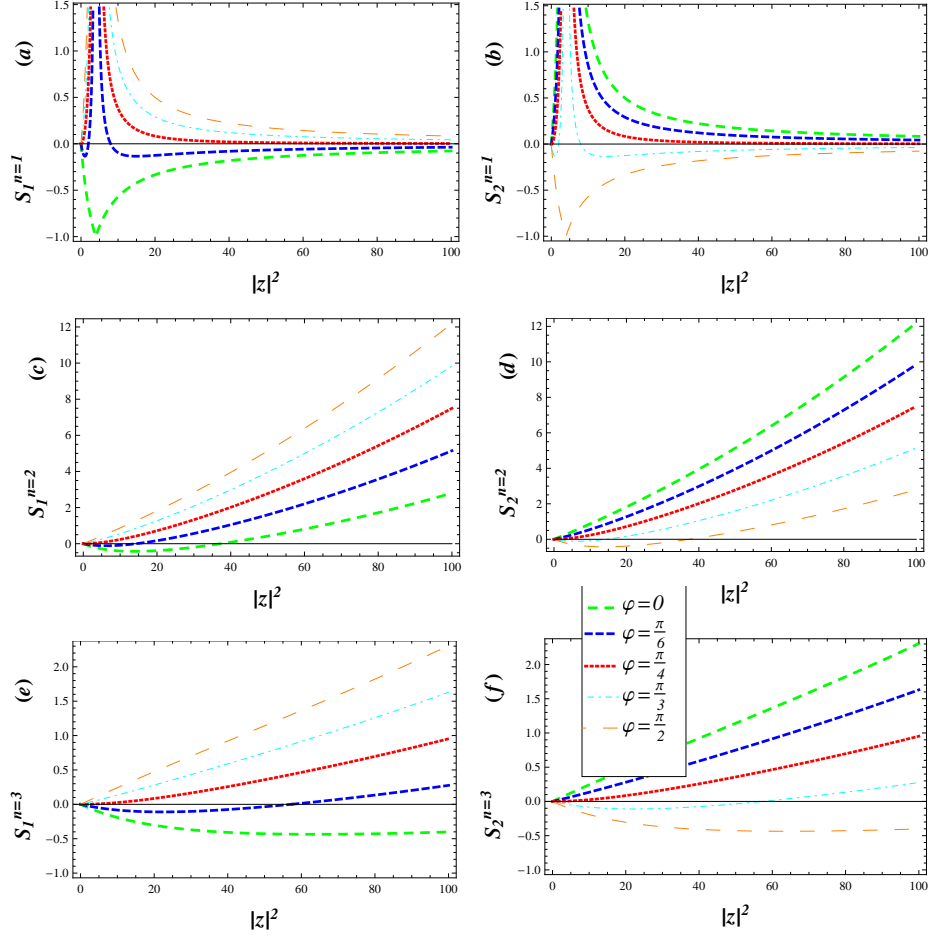


Figure 6: The plots of quadrature squeezing parameter S_1^n (a, c, e) and S_2^n (b, d, f) respectively, against $|z|^2$ for $r = 4$ and different values of n and ϕ .

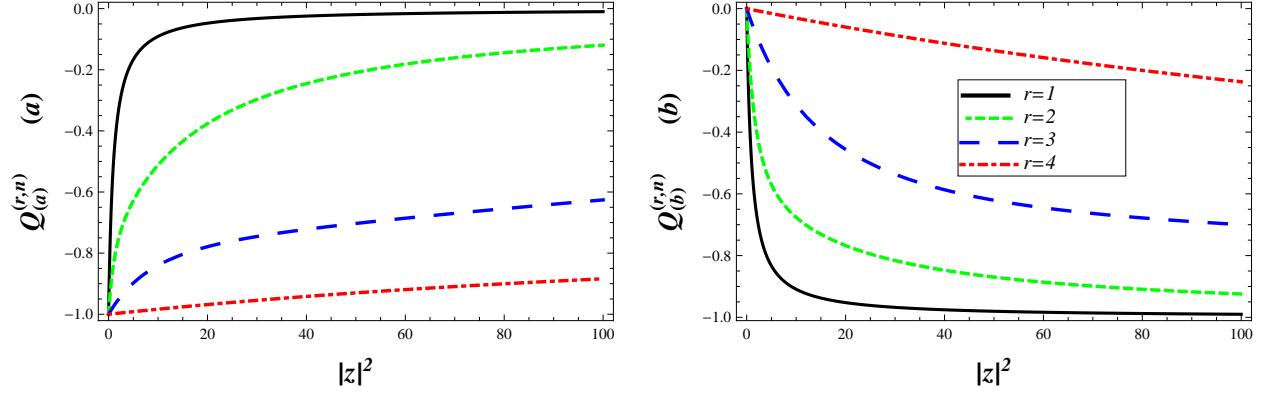


Figure 7: Plot of the Mandel parameters (a) $Q_a^{(r)}$ and (b) $Q_b^{(r)}$ versus $|z|^2$ for different values of r .

fields in the first and the second modes [68]

$$Q_i^{(r)}(|z|^2) = \frac{\langle \hat{N}_i^2 \rangle_r - \langle \hat{N}_i \rangle_r^2}{\langle \hat{N}_i \rangle_r} - 1, \quad i = (a \text{ or } b) \quad (59)$$

where, \hat{N}_i is photon number operator in first and second modes. In fact, a quantum state exhibits super-Poissonian (photon-bunching), Poissonian and sub-Poissonian (photon-antibunching) statistics, respectively, if $Q_i^{(r)} > 0$, $Q_i^{(r)} = 0$ and $Q_i^{(r)} < 0$. Now, evaluating the mean value of \hat{N}_i and \hat{N}_i^2 with respect to the states $|z\rangle_r$ yield

$$\begin{aligned} \langle \hat{N}_b \rangle_r &= \left(\frac{|z| \sqrt{n} \Gamma(n)}{\Gamma(n+r-1)} \right)^2 \\ &\times \frac{{}_1F_{2r-2}([1-n], [n+1, n+1, \dots, n+r-1, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2)}, \\ \langle \hat{N}_b^2 \rangle_r &= \left(\frac{|z| \sqrt{n} \Gamma(n)}{\Gamma(n+r-1)} \right)^2 \\ &\times \frac{{}_2F_{2r-1}([2, 1-n], [1, n+1, n+1, \dots, n+r-1, n+r-1], -|z|^2)}{{}_1F_{2r-2}([-n], [n, n, \dots, n+r-2, n+r-2], -|z|^2)}, \\ \langle \hat{N}_a \rangle_r &= n - \langle \hat{N}_b \rangle_r, \\ \langle \hat{N}_a^2 \rangle_r &= \langle \hat{N}_b^2 \rangle_r - 2n \langle \hat{N}_b \rangle_r + n^2. \end{aligned} \quad (60)$$

In Figure 7, the Mandel parameters of two modes, $Q_a^{(r)}$ and $Q_b^{(r)}$, have been plotted in terms of $|z|^2$ for $n = 2$ and different values of $r (=1, 2, 3 \text{ and } 4)$. As is evident, both modes have sub-Poissonian statistics for any values of generalization parameter r . From figure 7(a), in

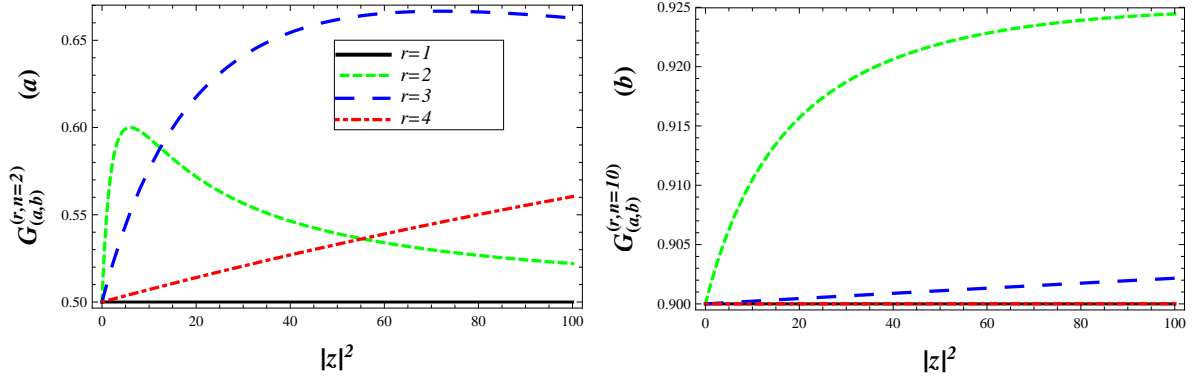


Figure 8: Plot of Cross-correlation function $G_r^{(2)}$ versus $|z|^2$ for different values of r , with (a) $n=2$ and (b) $n=10$.

the limit of $|z|^2 \rightarrow 0$, the Mandel parameter of first mode is $Q_a^{(r)} = -1$, which indicate that first mode is maximally sub-Poissonian. This situation is changed for second mode. As shown in figure 7(b), the second mode is maximally sub-Poissonian for $|z|^2 \rightarrow \infty$. Also, by increasing the generalization parameter r , photon statistics of the first (second) mode of the generalized CSs tends to sub-Poissonian (Poissonian) more rapidly. In other words, nonclassical properties of the first (second) mode of the generalized CSs increases (decreases) by increasing (decreasing) of generalization parameter r . Now we turn our attention to the anticorrelation properties of the fields under consideration. Anticorrelations are described by the normalized cross-correlation functions $G_r^{(2)}$:

$$G_r^{(2)}(|z|^2) = \frac{\langle \hat{N}_b \hat{N}_a \rangle_r}{\langle \hat{N}_b \rangle_r \langle \hat{N}_a \rangle_r}, \quad (61)$$

which by using Eq. (34) and the following equation

$$\langle \hat{N}_b \hat{N}_a \rangle_r = n \langle \hat{N}_b \rangle_r - \langle \hat{N}_b^2 \rangle_r, \quad (62)$$

we get

$$G_r^{(2)}(|z|^2) = \frac{n \langle \hat{N}_b \rangle_r - \langle \hat{N}_b^2 \rangle_r}{\langle \hat{N}_b \rangle_r (n - \langle \hat{N}_b \rangle_r)}. \quad (63)$$

Figure 6 displays the cross-correlation function for the $|z\rangle_r$, with respect to $|z|^2$ for $n = 2, 10$ and $r = (1, 2, 3 \text{ and } 4)$. From figure 8(a), for any value of r , cross-correlation function is less than one. This indicates that the two bosonic modes which are discussed are anti-correlated. Physically, this means that there is no tendency for bosons in the different modes to be created or annihilated simultaneously. Also, as shown in figure 8(b), for large values of n , $G_r^{(2)}(|z|^2)$ is equal to one, which indicates that the bosons in the different modes are created or annihilated independently.

5 Conclusions

Based on a new approach, broad range of generalized $su(2)$ CSs for problem of a charged particle moving on an infinite flat band in the presence of a constant magnetic field are constructed. We have shown that these states are nonlinear truncated CSs with a spacial nonlinearity function. Then, the nonclassical properties such as $su(2)$ squeezing, quadrature squeezing and sub-Poissonian statistics for the introduced states in addition to their anti-correlation have been reviewed, in detail. It has been shown that, they exhibit $su(2)$ squeezing in both components X_1 and X_2 . Also, from Figure 5 the quadrature squeezing only in the angular momentum component (p) for ($M = \omega = \hbar = 1$). This situation can be changed by increasing of external magnetic field B_{ext} . It is easy to show that the squeezing in p component can be transformed into x component by increasing B_{ext} and vice versa. In fact, the squeezing effect in the x and p component is flexible by variation of magnetic field. Our numerical results have explicitly shown that both modes have sub-Poissonian statistics for any values of generalization parameter r . Furthermore, increasing of r leads to the enhancement of nonclassical properties of the first mode and causes to diminish nonclassical properties of the second mode. Generally, the approach presented here can be used to construct new type generalized $su(2)$ CSs for exactly solvable models, such as two dimensional harmonic oscillator.

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